Coastal and Hydraulics Engineering Technical Note IV-__
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US Army Corps of Engineers

Uncertainty in Bathymetric Surveys

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PURPOSE: This Coastal and Hydraulics Engineering Technical Note (CHETN) presents a statistical methodology for assessing the uncertainty of bathymetric analyses. The uncertainty assessment is based on the reported accuracy of a bathymetric survey. The U.S. Army Corps of Engineers (USACE) regularly conducts surveys to evaluate the navigability of waterways and to determine the pay quantity after a dredging project. The aerial extent and volume of subaqueous features can also be estimated from bathymetric data.

BACKGROUND: The USACE maintains more than 12,000 miles of waterways (USACE 2002). Channel conditions and subaqueous features, such as an ebb-tidal shoal, are determined from hydrographic or bathymetric surveys.[1] Surveys may be conducted with standard single beam echo sounders or high-resolution multibeam echo sounders. Survey standards follow those outlined in the Engineering and Design - Hydrographic Surveying manual (USACE 2002). The dredged volume and pay quantity are determined from existing and post-dredging surveys.

Although individual points define the accuracy of a bathymetric survey, all points within the focus area determine the overall accuracy of bathymetric analyses describing geomorphic changes. In this CHETN, the uncertainty related to bathymetric analysis is addressed statistically with confidence intervals for elevation and volume. This methodology is not intended to assess the accuracy of hydrograph surveys performed to determine navigability. The Engineering and Design - Hydrographic Surveying manual (USACE 2002) provides an extensive discussion of technical guidance and accuracy standards for hydrographic surveys.

Geomorphic analysis of coastal environments typically involves describing elevation, slope, shape, and sediment volume of features on the seafloor. Advances in computer technology have automated quantification of such properties. Mathematical computations are often reported with sub meter precision. Although the calculation may have that precision, uncertainty in the raw data will lower and control the practical level of accuracy. Understanding uncertainty and reporting it contributes to reliability of the final conclusions. There are many sources of error inherent in calculating changes in sediment volume from bathymetric maps collected at different times. Sources of error are classified as blunders, systematic errors, and random errors (Mills 1998; USACE 2002). Blunders are human errors and can be minimized through care and attention during the survey and analysis. Systematic errors are caused by a condition that may be predicted and modeled, such as

[1] A hydrographic survey is performed to identify the limiting depth and hazards to navigation. A bathymetric survey is performed to determine the morphology or elevation of a specified area. In this CHETN, these survey types are treated similarly with regard to determining uncertainty.

inaccurate calibration of an instrument or a mis-located datum (Byrnes, Li, and Baker 2002). Repeated surveying on consecutive days over the same area may identify systematic errors introduced through tidal corrections (Hicks and Hume 1997). Once a systematic error is identified, it can be corrected. Random errors cannot be predicted, are generally small relative to the depth being measured, and may cancel during processing. However, the random error may be large compared to the elevation difference between two bathymetric surveys. Random errors may arise through the inability to perfectly measure the depth, natural variation in the seafloor at a spatial scale smaller than the sampling interval, or through interpretation methods. Random errors are assumed to be normally distributed and can be analyzed statistically (Mills 1998, USACE 2002).

In this CHETN, uncertainty refers to a quantity or process associated with the measurement or analysis that is not known, for example, interpolation between points. The interpolation may be correct, but data are not available to verify the values produced. Simply stated, an error is something that is wrong; for example, an elevation sample that is not reported correctly is considered to be wrong. The potential for introducing errors is greater in the marine environment than on land because the surveyor cannot see what is being surveyed, and there is uncertainty in the position of the survey vessel location (Byrnes, Li, and Baker 2002).

This CHETN addresses random vertical errors in bathymetric surveys, not systematic errors, blunders, or any type of horizontal error. Data density and interpolation methods will be evaluated in three different coastal environments—tidal inlet channel, ebb-tidal shoal, and offshore area. Channels are areas of high relief; ebb shoals have moderate relief, and the offshore has the least relief (is more smoothly varying).

Data: Data sets were selected to represent typical environments encountered in bathymetric analysis of coastal inlets. The data were collected by Scanning Hydrographic Operational Airborne LIDAR Survey (SHOALS) at Shinnecock Inlet in Southampton, NY in July 2000 and July 1994 (Figure 1). The survey data have a reported horizontal accuracy of ±3 m (1-sigma standard deviation) and the vertical accuracy is ±15 cm (1-sigma). These accuracies comply with the standards of the International Hydrographic Organization Order 1 (USACE 2002). The surveys were originally compiled in New York State Plane Feet, North American Datum of 1983, Long Island Zone, National Geodetic Vertical Datum of 1929. The coordinate system and datums were retained for present analysis, but the units were converted from feet to meters to conform to customary scientific format.

The data were subdivided into three smaller sub regions based on the physical characteristics of the site—the tidal inlet channel, ebb shoal, and offshore (Figure 2, Table 1). From each sub region, 5 percent of the data points were randomly removed from the data set to serve as verification points to compare the accuracy of the surface model to measured (true) elevations from the survey. The verification points provided ground-truth comparisons for the various interpolation methods and for identification of errors associated with data density. Any surface model created for these two analyses did not include the verification points. The verification points were not necessary for the statistical analysis of the instrument error; therefore, the verification points were included in the interpolation for Monte Carlo simulation.

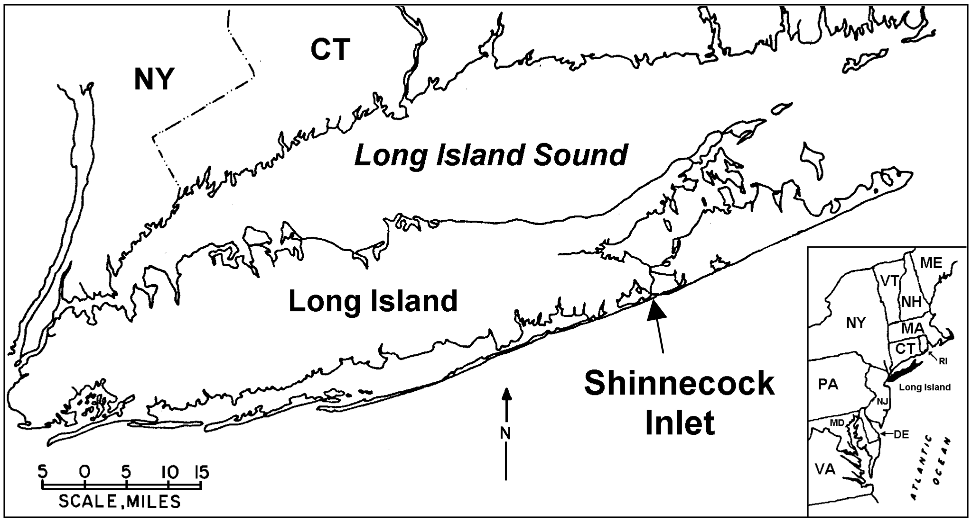

Figure 1. Regional map

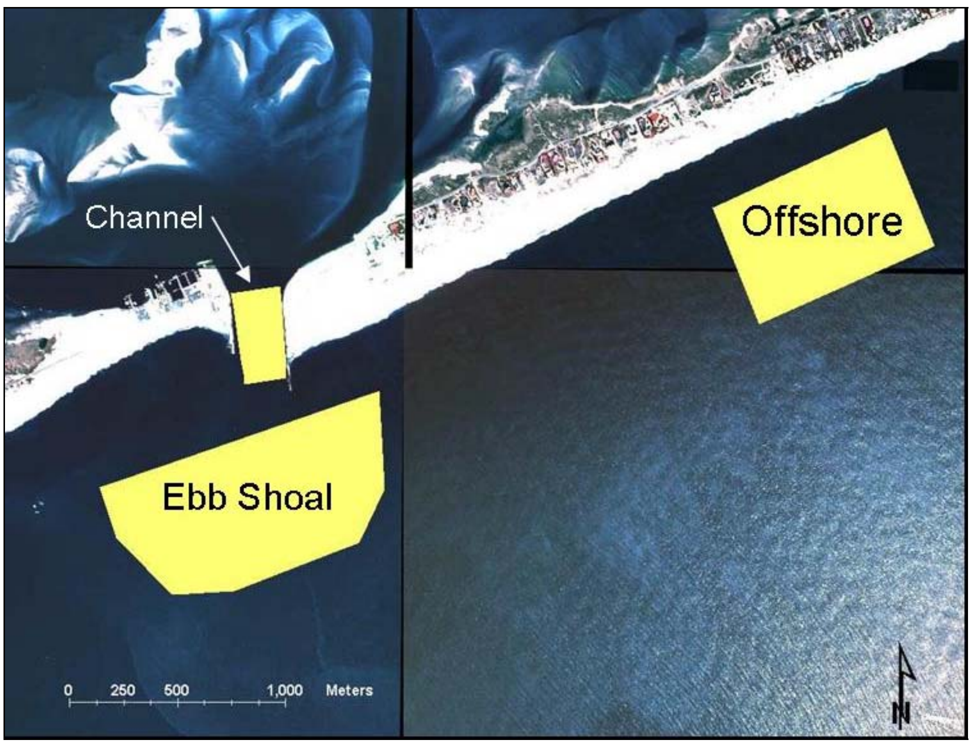

Figure 2. Location map of sub regions

Table 1 Data for Analysis				
Sub Region	**Area, m^2**	**Elevation Range, m**	**# of Verification Points (5 percent)**	**Density Pts/100 m^2**
Ebb Shoal	769,700	6.5	2437	6.3
Channel	91,350	17.3	229	5.0
Offshore	522,550	7.5	1615	6.2

INTERPOLATION ANALYSIS: It is necessary to interpolate survey data in order to produce a continuous bathymetric map. The accuracies of different bathymetric surfaces were determined through comparison of modeled elevations and measured depths (verification points) at corresponding locations. The bathymetric surface was interpolated with kriging and as a Triangulated Irregular Network (TIN).

Kriging is a geostatistical method for interpolating a surface from point data. Elevations are interpolated from the original data points to a regularly spaced grid. The interpolation is based on a global weighting scheme that is derived from the data variance. It is not limited to a linear weighting scheme. Variance, γ, is the average difference in elevation at a specific lag distance (Burrough and McDonnel 1998):

$$\tilde{a}(h) = \frac{1}{2n} \sum \{ z_i - z_{i-h} \}^2 \qquad (1)$$

where h is the lag distance, n is the number of observation pairs, z is the attribute of interest – in this case elevation, and i and i-h are index notations denoting the location of z. Here, lag distance is the distance between two points used to calculate the variance. A variogram is a plot of variance and the lag distance. There are two types of variograms – an experimental and a modeled variogram (Environmental Modeling Systems, Inc. 2002). The experimental variogram is the plot of the variance calculated at each lag distance. Once the experimental variogram is calculated, the modeled variogram can be represented with one of several equations, such as linear, spherical, Gaussian, exponential, or circular (Figure 3). The variogram model is used to interpolate the elevation at unknown points. The most accurate model will depend on the variance in the data. Three defining features of the variogram are the sill, range, and nugget. The sill describes the variance within the data at distances far enough apart that there is no spatial correlation. The range is the distance at which spatial correlation is no longer evident and the sill has been reached. The nugget is the variance at lag distances smaller than the sampling interval or noise in the data.

Bathymetric surfaces were created with six methods, five methods of kriging and a TIN. The five kriging methods includes a linear, spherical, exponential, Gaussian, and circular weighting scheme. The kriging was completed in the commercial software ArcINFO. Three different grid spacings (5, 10, and 15-m) were analyzed with each method for a total of 15 different kriged interpretations of the surface. The final method of interpolation was to create a TIN in ArcView. To compare elevation of the TIN and the verification points, it was necessary to convert the TIN to a gridded surface. The surface generated from the TIN had 5-m grid spacing.

Interpolation Analysis Results: The experimental variogram of each subarea is shown in Figure 4. In the channel, the sill is about 12 m, and the range is about 150 m. Offshore, the sill is about 7 m and the range is between 700 and 1,000 m, depending on the interpolation method. The sill and range are not clearly defined on the ebb shoal. The lack of a defined sill and range on the ebb shoal suggests that the entire shoal may exhibit some degree of spatial correlation.

The accuracy of the surface interpolation was determined by finding the difference between the elevation of the verification point and the elevation of surface model at the same location. The accuracy of the model was quantified with a linear regression and the 95 percent confidence interval of the errors. From each linear regression, the R^2 value was determined (Figure 5, Appendix 1).[2] All the R^2 values were high, indicating that all the interpolation methods worked well with this data set. The offshore areas were all above 0.998, the channel was above 0.93, and the ebb shoal was above 0.995. The high value and narrow range of the R^2 values indicate that all methods were essentially equivalent for this data set.

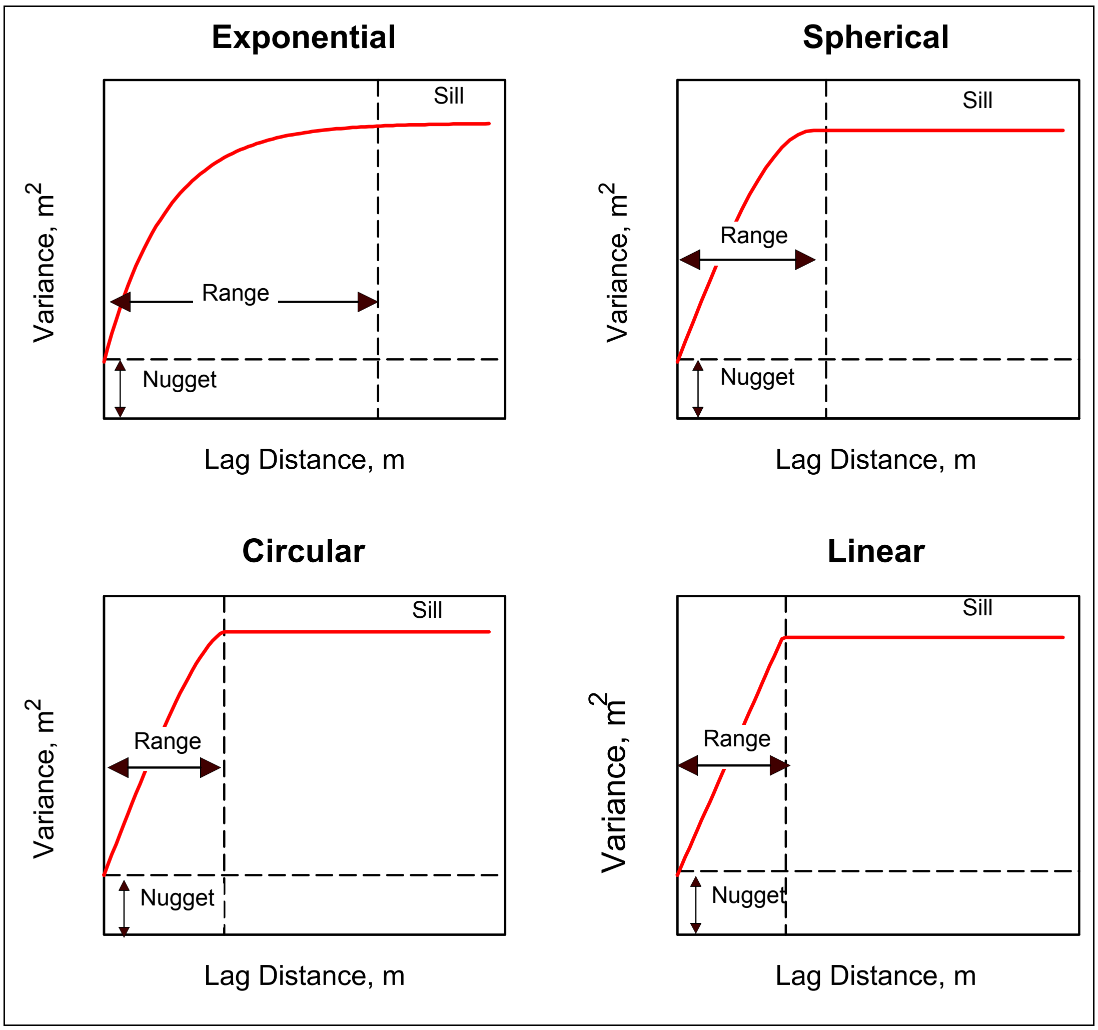

Figure 3. Example variograms using data from Shinnecock Inlet (after Burrough and McDonnel 1998)

[2] Only one example is shown here.

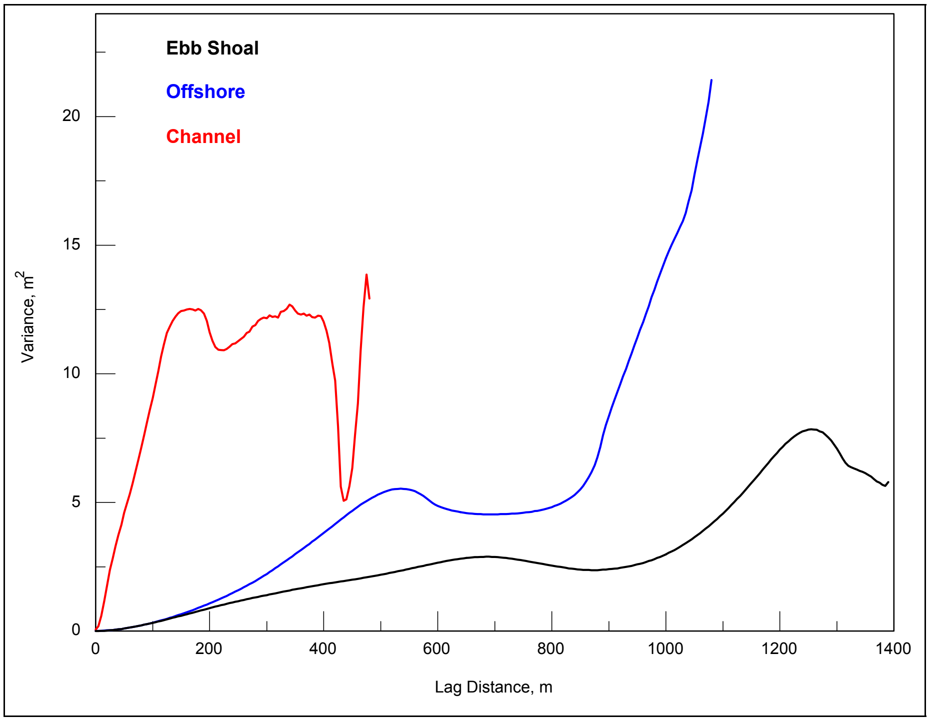

Figure 4. Experimental variogram for Shinnecock Inlet

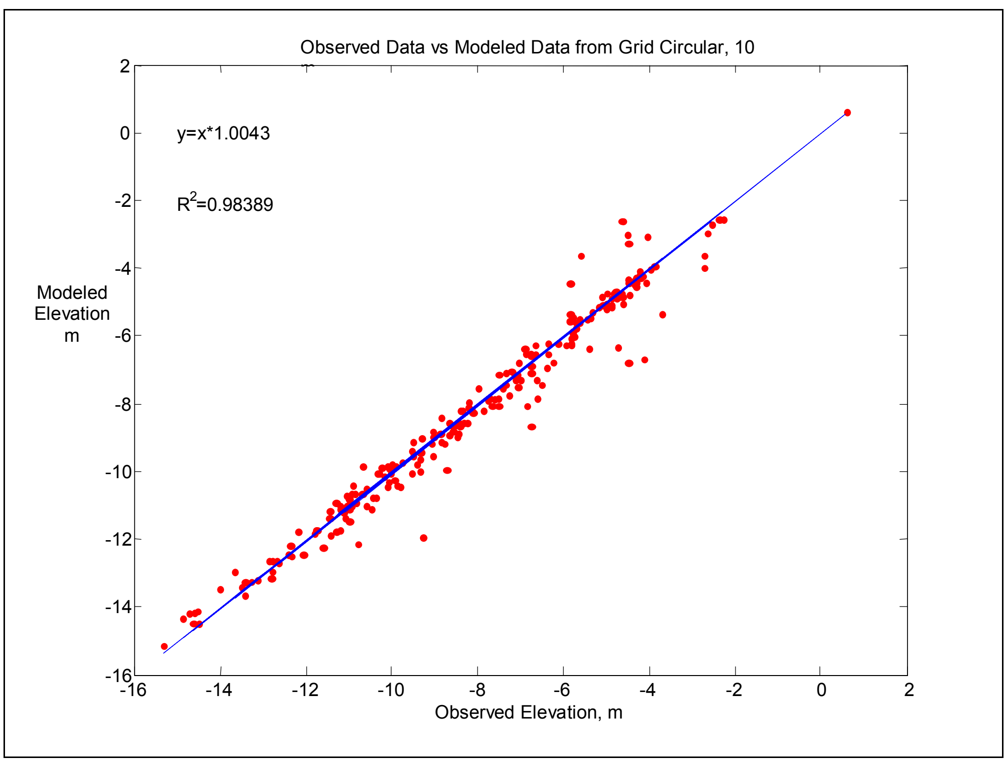

Figure 5. Example linear regression plot (channel, circular, 10-m grid spacing)

The next step in describing the accuracy of the modeled surface was to determine the elevation confidence interval for each surface model. The 95 percent confidence interval was calculated using the following equation (McClave and Sincich1997):[3]

$$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \tag{2}$$

where $\bar{x}$ is the mean, $z_{\alpha/2}$ is the z-value at a specified confidence level, s is the standard deviation, and n is the population size (number of verification points). For a confidence interval of 95 percent $z_{\alpha/2}$ is 1.96.

The 95 percent confidence intervals for 16 interpolations are shown on the bar graph in Figure 6. The top and bottom graphs are the same except for the scale of the y-axis. The y-axis on the bottom graph was expanded so that the confidence level for the ebb shoal and offshore can be seen. The reported SHOALS instrument error is ±15 cm. All confidence intervals are small compared to the instrument error. At Shinnecock Inlet, all of the interpolation uncertainty for the offshore area and the ebb shoal is less than 1 cm. The largest errors are within the channel if the 15-m grid was used, but these confidence intervals are still less than 15 cm. The cell spacing had more control over the confidence interval than the method of interpolation. The large cell spacing had more error. If the data points were sparser then the interpolation error would be expected to be larger.

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[3] All confidence intervals discussed in this CHETN are 95 percent.

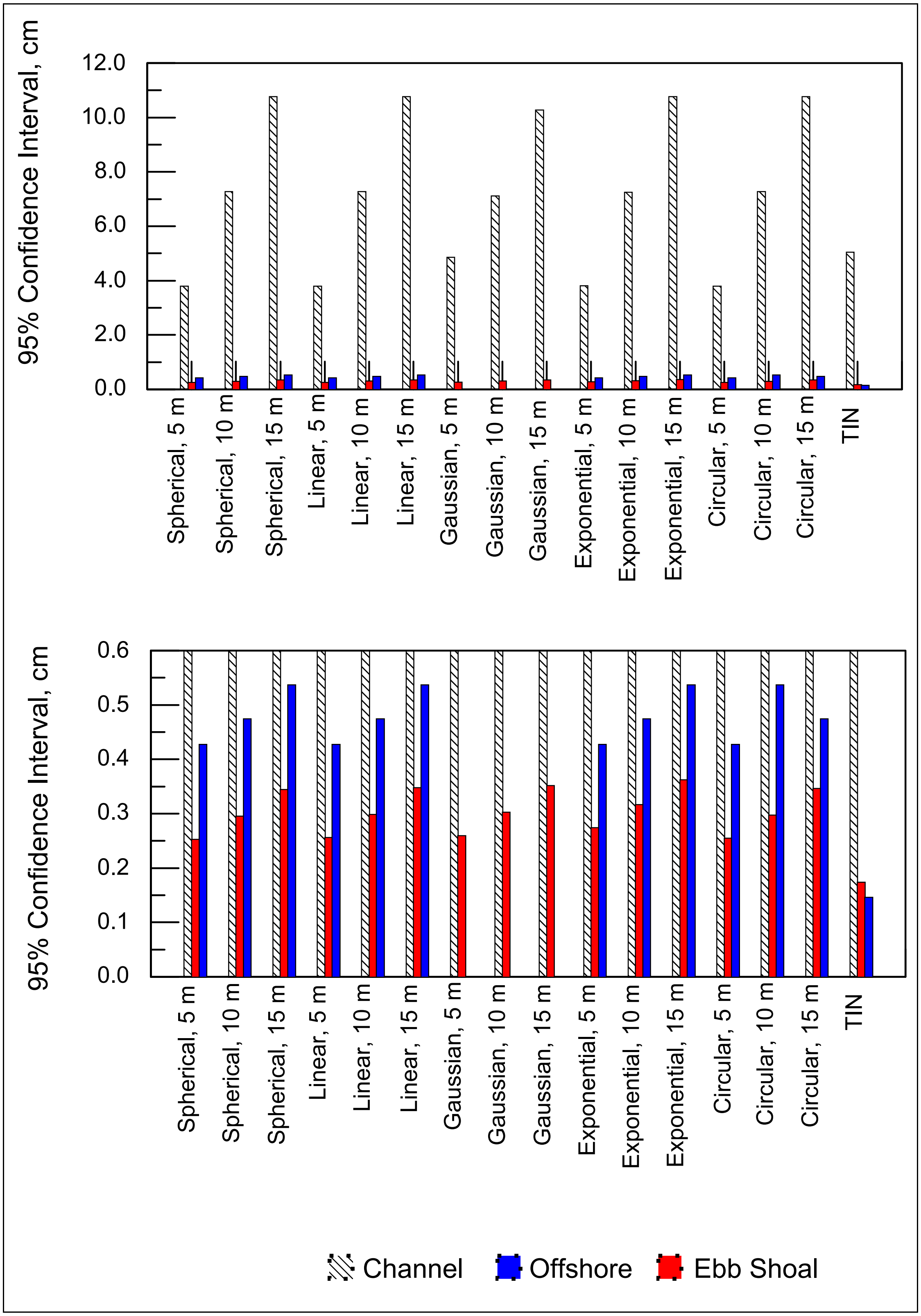

Figure 6. Confidence interval for original data

DATA DENSITY: The role of data density on uncertainty in bathymetric interpolation was examined by repeatedly thinning the data in each sub region. The thinned data sets were interpolated through kriging with the spherical weighting scheme and a 5-m cell spacing and as a TIN. The thinned data sets were not interpolated with all the methods described above because the degree of uncertainty was shown to be insensitive to the particular method. The surface interpolations for the density analysis were extrapolated to 5-m grid spacing because this spacing had the least uncertainty. Random thinning and a cross-channel transect sampling pattern created the subsets of the data for the density analysis. The data were randomly thinned to different densities ranging from 6 to 1/8 points/100 m^2. The cross-channel transect survey was based on transects is shown in Figure 7. A cross-channel transect sampling pattern is common in channel surveys. Transects are generally spaced 50, 100, or 200 ft apart (USACE 2002). This approach was originally implemented in an era when volume calculations were done by hand based on the average end area method. Transects in this example were 33 m apart, whereas the data points along the transects were about 5 m apart. The channel was the only subarea where transects were created.

Data Density Results: The confidence intervals calculated for the ebb shoal and the offshore area, at varying densities, are similar in magnitude (Figure 8). The confidence intervals for the ebb shoal are smaller than those of the offshore area for high point densities. Statistics for the channel show larger confidence intervals than the other two areas. This trend may be attributed to the greater elevation range in the channel. At the lowest data densities, less that 0.5 points/100 m^2, the interpolation error approaches the instrument error (±15 cm) in the channel. If the density was above 2 points/100 m^2, the confidence interval for the elevation did not decrease significantly. For the cross-channel transect survey, the 95 percent confidence intervals were 6 and 7 cm, for the TIN and kriged models, respectively.

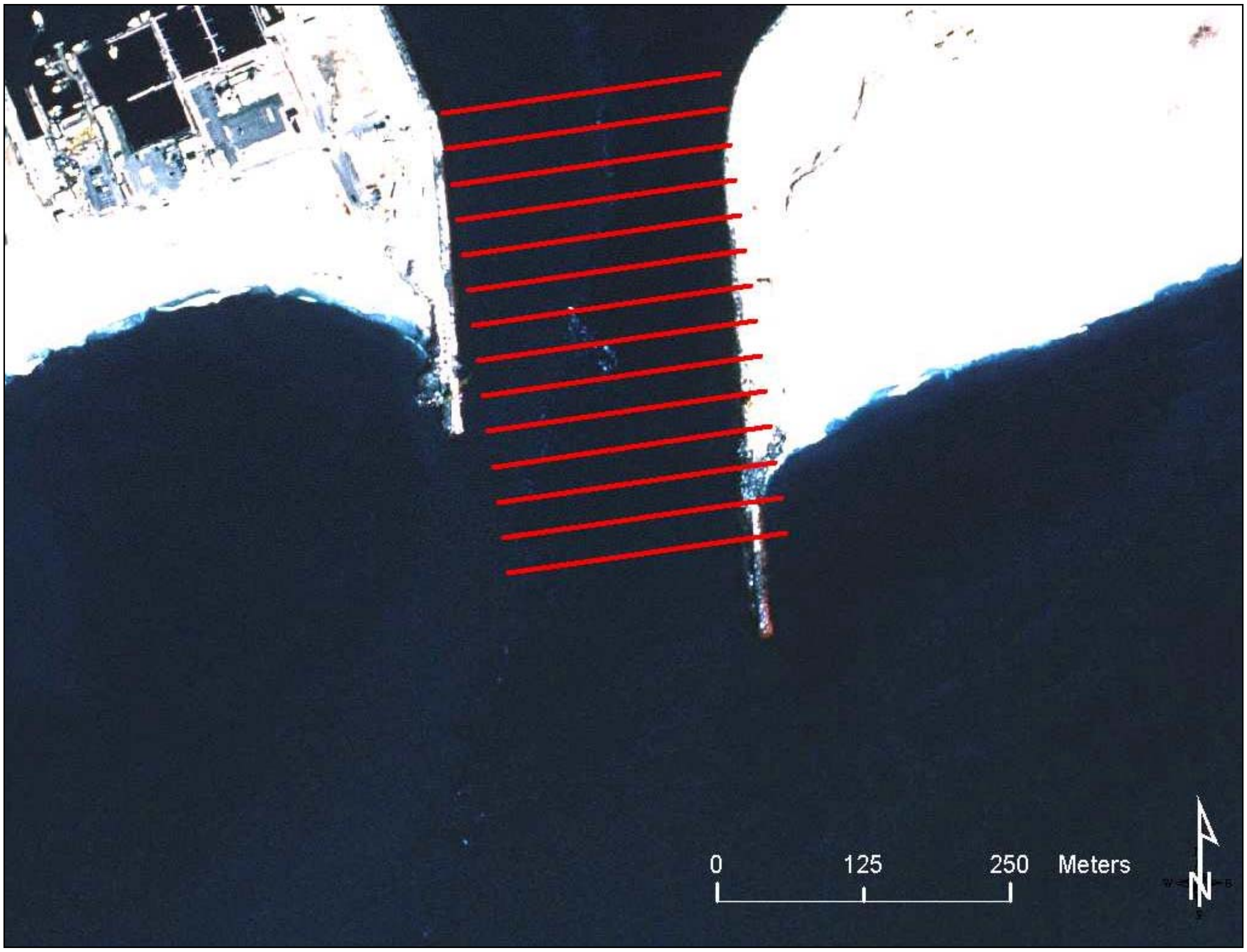

Figure 7. Cross-channel transects

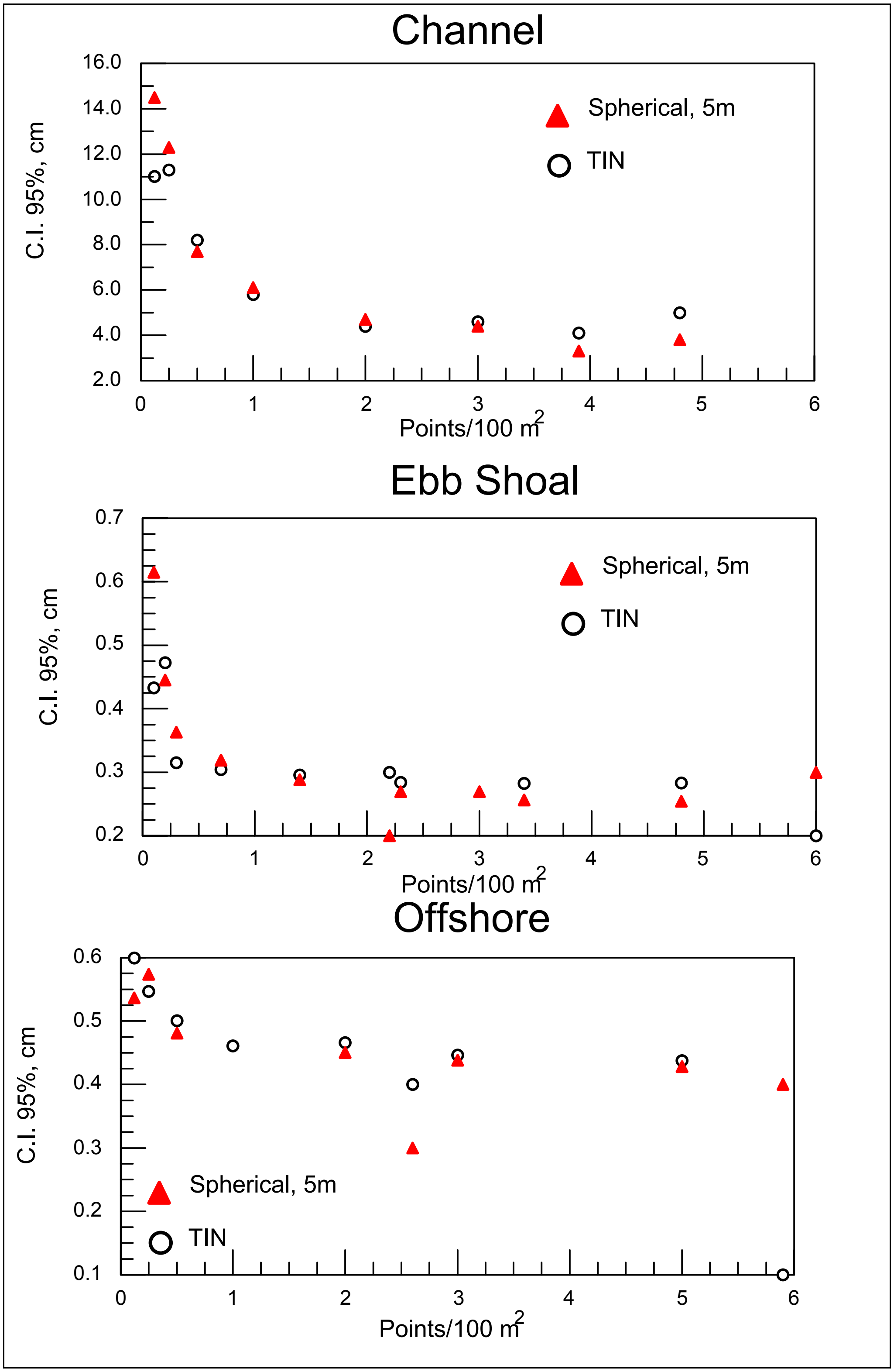

Figure 8. Confidence interval vs. point density

INSTRUMENT ERROR: The stated instrument error for the 2000 SHOALS data set at Shinnecock Inlet is ±15 cm. Because this is a random error, its contribution to the uncertainty will depend on the type of question to be answered. The accuracy of a single point is ±15 cm, so if the elevation at one location is being examined, then the error can also be reported as ±15 cm. However, if the parameter to be estimated involves multiple data points, such as estimating an average depth, slope, or volume, the instrument error should be evaluated statistically. In the following section, a Monte Carlo simulation is used to statistically examine volume calculations and identification of areas of erosion/shoaling.

Volume: The volume for each sub area was estimated 100 times in the commercial available software package MatLab (Appendix 2). Each estimate changed because a random error within ±15 cm was added to each data point, and therefore the elevations in each calculation changed slightly. A 95 percent confidence interval was calculated from the resulting 100 volume estimates. The number of volume calculations was limited by computer capability. One hundred trials were sufficient because additional calculations no longer changed the confidence interval significantly (Figure 9). This calculation was completed at different point densities. If less than 100 percent of the data points were selected for analysis, different points were selected each time. For example, one run used 70 percent of the original data set, so for each volume calculation a new subset of the data was selected with70 percent of the original points.

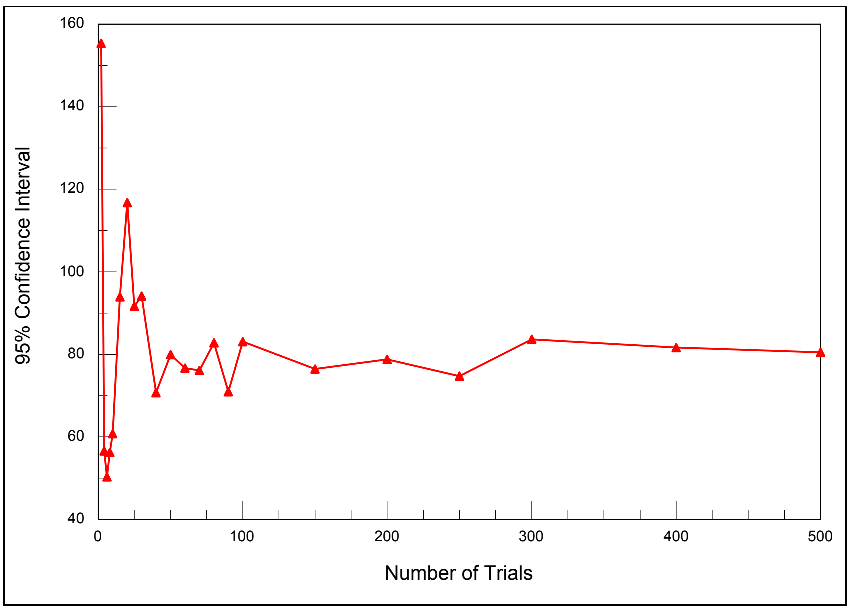

Figure 9. Confidence interval vs. number of volume calculations

The confidence interval and average volume are plotted against data density (Figure 10). The confidence interval for the channel, offshore, and the ebb shoal decreased steeply from 1 to 3 points/100 m^2. Data densities higher than 3 points/100 m^2 improved the volume estimate only slightly. The channel transect survey had a density of less than 1 point/100 m^2, but had a confidence interval similar to that obtained at 4 points/100 m^2 with random thinning. The confidence interval for the transect may be deceivingly small because the same points entered every calculation. The only variation in the volume calculation was in the random error (±15 cm).

The average volume for the ebb shoal, channel, and offshore sub regions increases as the number of points included in the calculation increase. At point densities greater than 2 points/100 m^2, the average volume levels off for each of the sub regions. The decrease at low point densities occurs if points along the edge of the sub region are not included. This will cause the area of the volume calculation to decrease and therefore the volume will also decrease.

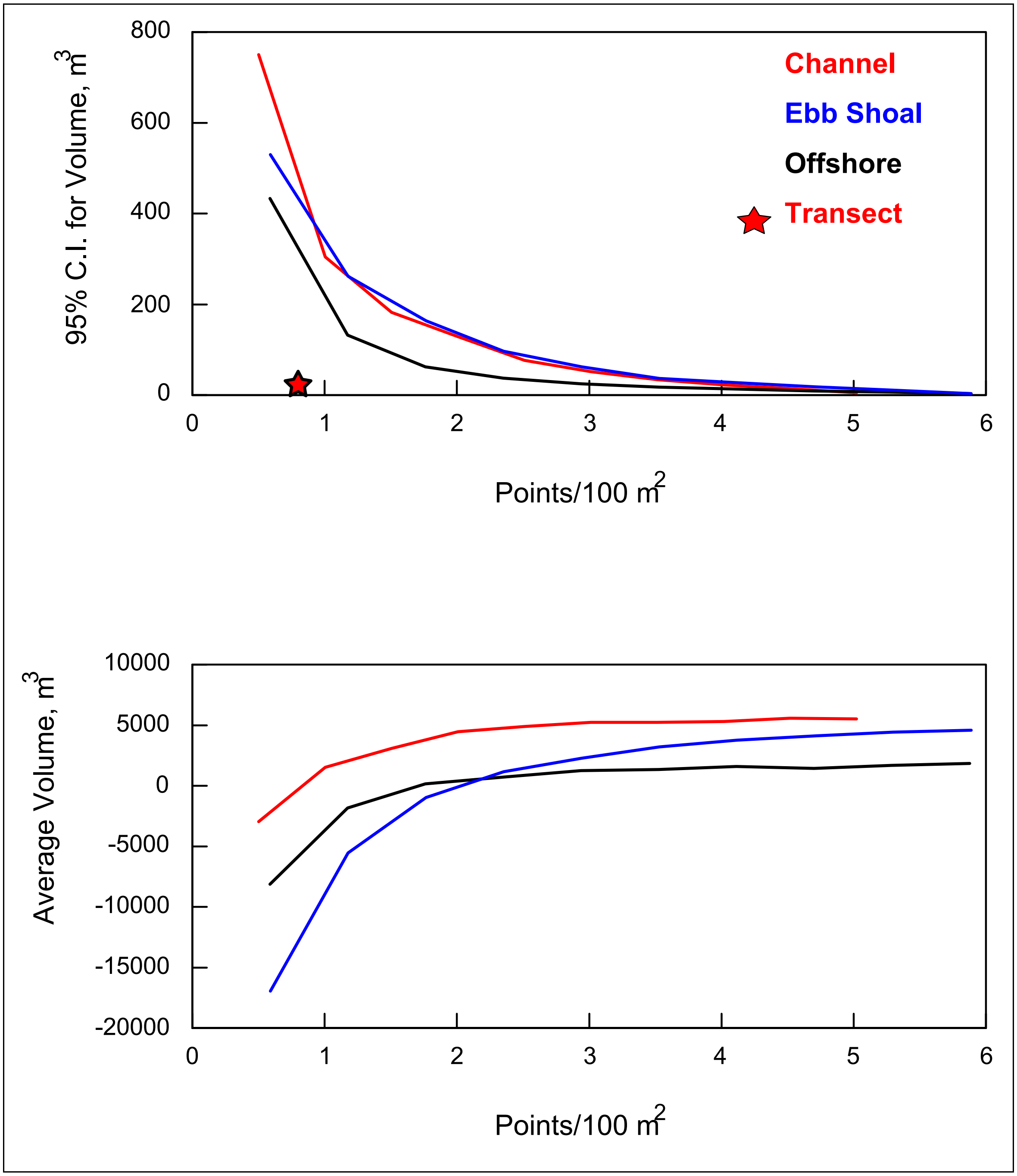

Figure 10. Results of volume analysis

Erosion/Shoaling: This section describes the uncertainty in identifying areas of erosion and accretion with a fuzzy set (Burrough and McDonnel 1998). A fuzzy set allows the entities (in this context, the grid cells), to belong to more than one thematic class. Here, each cell can represent erosion, accretion, or erosion and accretion. The degree of membership to each class is an indication of the uncertainty. The more traditional crisp set classification requires each entity be assigned to just one class. Because locating regions of erosion/accretion involves two surveys, the uncertainty of the results will be greater than the uncertainty associated with the individual surveys. An uncertainty array was calculated in MatLab (Appendix 3). Both surveys were loaded into the computer, and an "if" statement was evaluated at each grid cell. If the year 2000 survey was above the year 1994 survey, the uncertainty value was set to 1; if not, the value was 0. This process was repeated 50 times, with a new random error being added to both surveys each time. The new uncertainty values were added to the previous values. In the end, each cell had a value ranging from 0 to 50. If a cell always showed accretion, the uncertainty value was 50, whereas if it was always erosion the value was 0. The values of 50 and 0 identify areas where change in elevation was certain. There intermediate values showed areas of uncertainty (Figure 11).

The resulting plot expresses the degree of uncertainty of the erosion/accretion analysis. The user can then decide what accuracy is necessary for the application at hand. This method has been applied in other applications as well. Woodcock and Gopal (2000) described the uncertainty associated with the Plumas National Forest land cover data set using fuzzy sets. Areas receiving the "absolutely right" description represent the upper confidence area, and the other descriptions had lower confidence.

The area of erosion and accretion was estimated at each confidence level. The results showed the area of each decreased as the confidence level increased (Figure 12). The smallest areas were calculated at the "certain" level and the areas of each class increased as the confidence of the classification decreased. At low levels of certainty the sum of the area of erosion and accretion is greater than the total area of the channel. As uncertainty increases, the area belonging to both classes increases. Cells with membership in both classes, erosion and accretion, are accounted for twice in the area calculation. Calculations based on a low degree of certainty will tend to overestimate the area, whereas a high degree of certainty will underestimate the area.

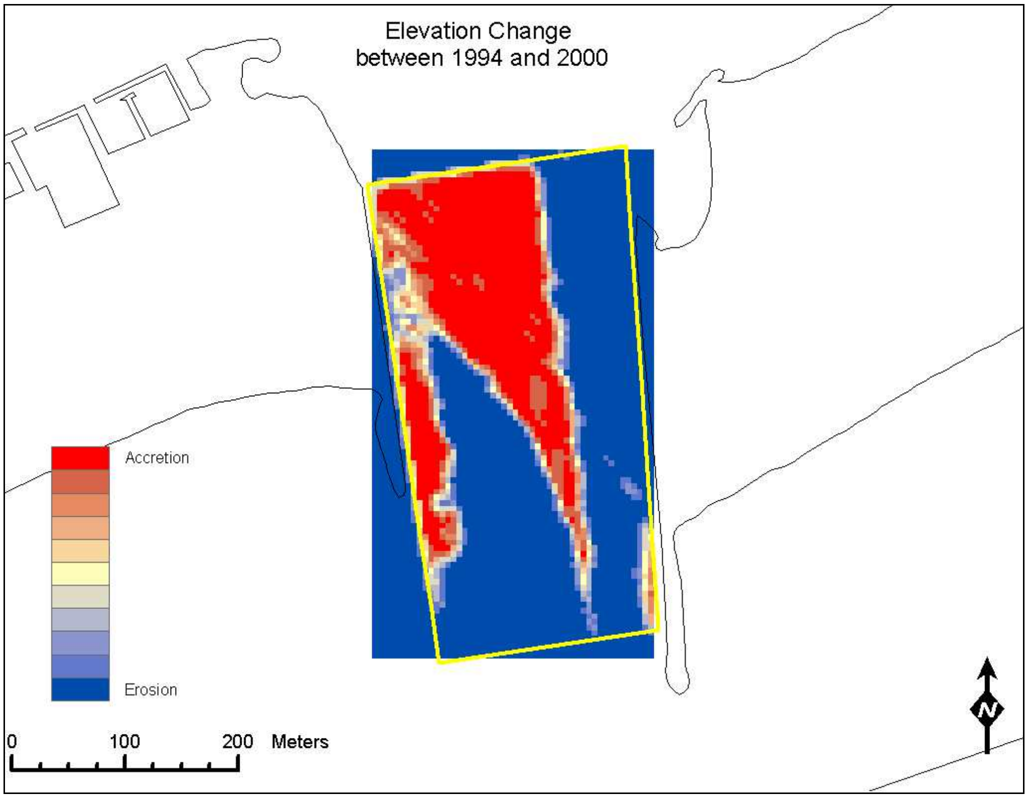

Figure 11. Map of erosion/accretion uncertainty

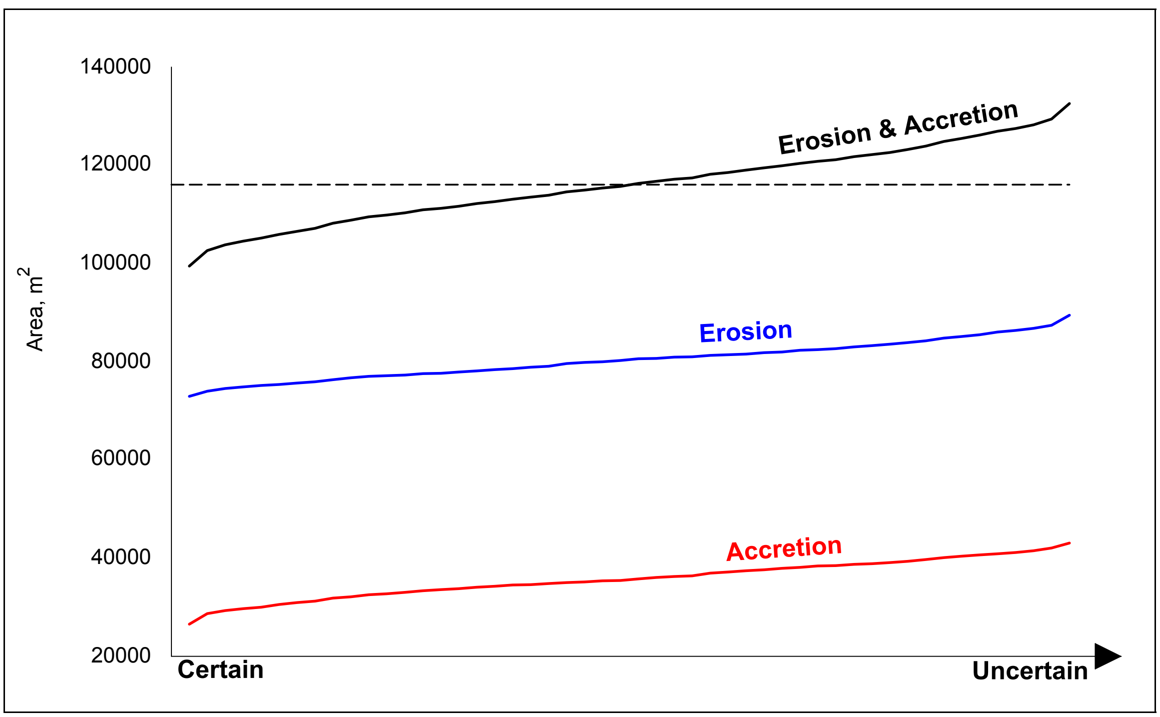

Figure 12. Area vs. uncertainty

CONCLUSIONS: The accuracy of bathymetric analyses can be statistically estimated based on the uncertainty of a single data point. At Shinnecock Inlet, the instrument error of the survey was determined to be greater than the interpolation error, at point densities greater than 2 points/100 m^2. Increasing data density can reduce the uncertainty. Although calculations described in this CHETN show that relatively low data density provide reasonably accurate volume calculations, the scale of interest must be considered. The smallest area considered here was a navigation channel at a coastal inlet. If a smaller area such as an individual shoal is the focus of the project, higher data density may be required.

Although the results presented in this technical note are specific to Shinnecock Inlet, the same method for evaluating uncertainty in bathymetric analyses can be applied to other areas. Procedures with general applicability that can be followed in other areas are:

a. Examine the uncertainty associated with the interpolation method though comparison of modeled elevations and the elevation at corresponding verification points.

b. Determine whether the interpolation or instrument error is greater.

c. Use the larger of the two errors in a Monte Carlo simulation to estimate a confidence interval for the volume calculation or identification of areas of erosion/accretion.

Based on the R^2 values of the linear regression and the 95 percent confidence intervals of the elevation difference between the actual and model elevations, it is concluded that interpolation methods and cell size have no significant effect on uncertainty. The instrument error was larger than the error of any interpolation process evaluated. The TIN method for creating bathymetric surfaces is recommended because it preformed as well or better than kriging. A TIN surface is also more likely to maintain the highs and lows present in the data than the kriging methods. Kriging inherently smoothes the data. Highs and lows are often the most important areas in bathymetric studies and are of central interest if a channel will be or has been dredged.

To increase the efficiency of computer analysis, it may be convenient to thin large data sets. In the case study completed for Shinnecock Inlet, thinning the data did not increase the interpolation uncertainty until the data density was less than 2 points/100 m^2. Transect surveys performed across the channel or perpendicular to the shoreline maintain higher accuracy at similar data densities than random surveys. This is because coastal environments display more variation in the cross-shore direction than the along-shore direction.

At high data densities, the uncertainty associated with interpolation is smaller than the instrument error. Although the elevation error at any location may be as much as 15 cm, the cumulative effect on an entire survey is small because the error is random. Random errors can be estimated with a Monte Carlo simulation. The uncertainties of the volumes calculated at Shinnecock were less than 1 percent of the total volume.

All of the analyses suggest that, in this example at Shinnecock Inlet, 2-3 points/100 m^2 is the lowest acceptable data density. At lower densities, the interpolation and volume confidence intervals begin to increase (Figure 8 and Figure 10).

The process presented in this CHETN to describe uncertainty in bathymetric analyses can be applied at other areas to calculate site-specific estimations. Reporting bathymetric change with an associated confidence interval is beneficial because confidence intervals are a standard statistical concept to document the degree of uncertainty (or reliability) of a data set.

ADDITIONAL INFORMATION:

This study is a product of the "Inlet Geomorphology and Channel Evolution" and "Inlet Channels and Adjacent Shorelines" Work Units of the Coastal Inlets Research Program (CIRP) being conducted at the U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory. This CHETN was written by Ms. Shelley Johnston, Graduate Research Student, Boston University. Questions about this CHETN can be addressed to Ms. Johnston at shelleyj@bu.edu or Dr. Nicholas C. Kraus, CIRP Program Manager, at Nicholas.C.Kraus@erdc.usace.army.mil. Beneficial reviews of this CHETN by Dr. Nicholas C. Kraus, Ms. Julie D. Rosati, Dr. Duncan FitzGerald, and Dr. Mark R. Byrnes are acknowledged with appreciation.

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Appendix 1
Regression Plots

```
% Evaluation of error between modeled surface and verification
points.
% The R squared value is calculated for the observed/modeled linear
regression.

%load data and set variables
clear;
close all;
load ('channel.txt');
obs=channel(:,3);
s5=channel(:,5);
s15=channel(:,6);
s10=channel(:,7);
l5=channel(:,8);
l15=channel(:,9);
l10=channel(:,10);
g5=channel(:,11);
g15=channel(:,12);
g10=channel(:,13);
e5=channel(:,14);
e15=channel(:,15);
e10=channel(:,16);
c5=channel(:,17);
c15=channel(:,18);
c10=channel(:,19);
var=[s5 s15 s10 l5 l15 l10 g5 g15 g10 e5 e15 e10 c5 c15 c10];
GD=['Grid Spherical, 5 m   ';
    'Grid Spherical, 15 m  ';
    'Grid Spherical, 10 m  ';
    'Grid Linear, 5 m      ';
    'Grid Linear, 15 m     ';
    'Grid Linear, 10 m     ';
    'Grid Gaussian, 5 m    ';
    'Grid Gaussian, 15 m   ';
    'Grid Gaussian, 10 m   ';
    'Grid Exponential, 5 m ';
    'Grid Exponential, 15 m';
    'Grid Exponential, 10 m';
    'Grid Circular, 5 m    ';
    'Grid Circular, 15 m   ';
    'Grid Circular, 10 m   '];

for i=[1:length(var)]
    y = var(:,i);
```

```
    x=obs;
    a(i) = x\y;
    res(:,i) = x*a(i);

%Calculate the R-squared,
    xmean=mean(x);
    ymean=mean(y);
    xdif=x-xmean;
    ydif=y-ymean;
    xy=xdif.*ydif;
    R2(i)=sum(xy)/(sum(xdif.^2));

%Plot
    figure(i)
    plot(obs,var(:,i),'.r');
    hold on;
    plot(obs,res(:,i),'b');
    text(-15,0,['y=x*',num2str(a(i))])
    text(-15,-2,['R^2=',num2str(R2(i))])
    title(['Observed Data vs Modeled Data from ',GD(i,:)]);
    xlabel('Observed Elevation, m');
    ylabel('Modeled Elevation, m');
end
```

Appendix 2
MatLab Program for Volume Confidence Interval

```
%Calculate a confidence interval for channel based on randomly
%selected percent
%of the origingal data.

%load data and set variables
clear;
close all;
load ('channel_org.txt');

l=length(channel_org);
trial=100;
percent=.5;           %Percent, as a decimal, of original data to

                      %include in the calculation.
n=round(l*percent);
z_error=zeros(n:trial);
d=5;                     %cell size
a=d*d;                   %m^2, area

%Start loop
for i=[1:trial]
%Choose Data Point
p = randperm(l);
p=p(1:n);
x=channel_org(p,1);
y=channel_org(p,2);
z=channel_org(p,3);

%Create grid lattice
xr=[min(x):d:max(x)];
yr=[min(y):d:max(y)];

%Create array of elevation plus a random error +-0.5 m
for j=[1:n]
    error=rand*.3;
    z_error(j,i)=(z(j)-0.15)+error;
end

%Create Grid
    [XI,YI] = meshgrid(xr,yr);
```

```
    ZI = griddata(x,y,z_error(:,i),XI,YI);
    %Calculate Volume
    %Interpolate cell value
    [r,c]=size(ZI);
    for q=[1:r-1]
        for w=[1:c-1]
         z_interp(q,w)=[ZI(q,w)+ZI(q+1,w)+ZI(q,w+1)+ZI(q+1,w+1)]/4;
        end
    end
    z_interp(isnan(z_interp))=-17;
    v(i)=sum(sum((z_interp-(-17))*a));
end     %End Start Loop

%Calculate 95% Confidence Interval
avg50=mean(v);
stdev=[(sum((v-avg50).^2))/(trial-1)]^0.5;
ci50=1.96*(stdev/(n^0.5));
```

Appendix 3
MatLab Program for Erosion/Shoaling Uncertainty

```
%Create a fuzzy set of erosion and accretion in Shinnecock Inlet

%load data and set variables
clear;
close all;
load ('channel_org.txt');
load ('Shin_1994.txt');

l=length(channel_org);
trial=5;
percent=.6;                %Percent, as a decimal, of original data
to include in the calculation.
n=round(l*percent);
d=5;                       %cell size

%Start loop
for i=[1:trial]
%Choose Data Point
p = randperm(l);
p=p(1:n);
x1=Shin_1994(p,1);
y1=Shin_1994(p,2);
z1=Shin_1994(p,3);
x2=channel_org(p,1);
y2=channel_org(p,2);
z2=channel_org(p,3);
x=[x2 x1];
y=[y1 y2];

%Create grid lattice
xr=[min(x):d:max(x)];
yr=[min(y):d:max(y)];

%Create array of elevation plus a random error +-0.5 m
for j=[1:n]
    error=rand*.3;
    z_error1(j,i)=(z1(j)-0.15)+error;
    error=rand*.3;
    z_error2(j,i)=(z2(j)-0.15)+error;
end
```

```
%Create Grid
[XI,YI] = meshgrid(xr,yr);
ZI1 = griddata(x1,y1,z_error1(:,i),XI,YI);
ZI2 = griddata(x2,y2,z_error2(:,i),XI,YI);
[r,c]=size(ZI2);

    if (i==1)
        for q=[1:r]
            for w=[1:c]
                if ZI1(q,w)<ZI2(q,w)
                     accr(q,w)=1;
                else accr(q,w)=0;
                end
            end
        end
    end

    if (i>1)
        for q=[1:r]
            for w=[1:c]
                if ZI1(q,w)<ZI2(q,w)
                   accr(q,w)=1+accr(q,w);
                else accr(q,w)=0+accr(q,w);
                end
            end
        end
    end

end     %End Start Loop

%Export Data
temp=0;
A = zeros(r*c,1);
for q=[1:r]
    for w=[1:c]
        temp=temp+1;
        A(temp,1)=XI(q,w);
        A(temp,2)=YI(q,w);
        A(temp,3)=accr(q,w);
    end
end
dlmwrite('my_data.out',A, ';')
```